

Anomalies, RG-flows and Open/Closed String Duality

Massimo Bianchi ¹ and Jose F. Morales ²

*D.A.M.T.P. University of Cambridge, UK
Dipartimento di Fisica and sez. I.N.F.N.,
Università di Roma “Tor Vergata”, Italia¹*

Spinoza Institute, Utrecht, The Netherlands ²

Abstract

We discuss the interplay between IR and UV divergences in vacuum configurations with open and unoriented strings. We establish a general one-to-one correspondence between anomalies and R-R tadpoles associated to sectors with non-trivial Witten index. The result does not require any supersymmetry to be preserved by the configuration. Under very mild conditions of supersymmetry, a similar correspondence is found between NS-NS tadpoles and RG-flows in gauge theories on D-branes and O-planes. We briefly comment on the AdS/CFT counterpart of the results.

MG9 Proceedings: submitted to World Scientific on January 12, 2001.

Keywords: Anomalies, Tadpoles, RG-flows, Open/Closed string duality

¹M.Bianchi@damtp.cam.ac.uk

²morales@phys.uu.nl

1 Introduction

The interplay between gauge theories and (super)gravity is one of the central issues in the string theory. In the past few years, some remarkable forms of duality between the two have been conjectured by BFSS [1] and Maldacena [2]. In our view, the duality between open and closed string channels offers the cleanest way to understanding the relation between gauge theories governing the low energy dynamics of configurations of D-branes and O-planes and certain (super)gravity theories living in the bulk of spacetime. A generic string amplitude around such backgrounds can be either thought in terms of closed string exchange between boundary and crosscap states or in terms of open string loops. In the limits where the worldsheet degenerates into long closed-string tubes or into thin open-string ribbons, only massless states contribute in either description and we can effectively interpret the results in terms of gauge theory or of supergravity. Of course, the two answers will in general disagree since they correspond to rather different truncations of the complete result. Under special conditions, however, they can be shown to coincide. In [3] these ideas were exploited in the context of Matrix theory. The leading order spin interactions between slowly moving parallel D-branes were extracted from fullfledged one-loop (in the open string channel) or tree level (in the closed string channel) amplitudes. The same result was shown to admit equivalent super Yang-Mills and supergravity descriptions. Aim of this talk is to review the correspondence between one-loop IR divergences, such as those giving rise to anomalies and RG-flows in gauge theories on D-branes and O-planes, and UV divergences associated to tadpoles in their dual supergravity descriptions. A complete exposition and references can be found in [4, 5]. We follow the notations and techniques developed in [6]. We will denote by \mathcal{K} , \mathcal{A} and \mathcal{M} , respectively, the contributions of Klein bottle, Annulus and Moebius strip to the relevant string amplitudes. By $\tilde{\mathcal{K}}$, $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{M}}$ we denote similar expressions rewritten in the closed string channel. The two sets are mapped into one another by (model dependent) modular transformations that exchange σ and τ directions on the worldsheet. At first sight one may thus in general doubt the existence of any connection between the truncations to their massless contributions of either description. A careful look into the structure of certain string amplitudes reveals that this is not always the case. In some cases, much as in the Matrix model computations, the zero-mode contribution happens to be exact and both descriptions turn out to be accurate.

2 Anomalies and R-R tadpoles

Let us consider a generic vacuum configuration containing open and unoriented strings. It can be specified in terms of its one-loop partition function³ ($\int \frac{dt}{t}$)

$$\begin{aligned}\mathcal{K} &= \frac{1}{2t^{\frac{D}{2}}} K^i \mathcal{X}_i(2it) \\ \mathcal{A} &= \frac{1}{2t^{\frac{D}{2}}} A_{ab}^i n^a n^b \mathcal{X}_i\left(\frac{it}{2}\right) \\ \mathcal{M} &= \frac{1}{2t^{\frac{D}{2}}} M_a^i n^a \hat{\mathcal{X}}_i\left(\frac{it}{2} + \frac{1}{2}\right)\end{aligned}\tag{2.1}$$

with \mathcal{X}_i a basis of charaters in the SCFT, a, b running over the number of boundaries, *i.e.* independent Chan-Paton charges, with integer multiplicities n^a . The integers K^i, A_{ab}^i, M_a^i count the number of times the sector i flows in the Klein bottle, Annulus and Möbius-strip loop, respectively⁴. In terms of closed string variables one can alternatively write ($\int dq$)

$$\begin{aligned}\tilde{\mathcal{K}} &= \frac{2^{\frac{D}{2}}}{2} \sum_i (\Gamma^i)^2 \mathcal{X}_i(q) \\ \tilde{\mathcal{A}} &= \frac{2^{-\frac{D}{2}}}{2} \sum_{i,a} (B_a^i n^a)^2 \mathcal{X}_i(q) \\ \tilde{\mathcal{M}} &= \frac{2}{2} \sum_{i,a} (\Gamma^i B_a^i n^a) \hat{\mathcal{X}}_i(-q)\end{aligned}\tag{2.2}$$

The relative powers of 2 result from the different rescalings of the modular parameters ($\tau_{\mathcal{K}} = 2it, \tau_{\mathcal{A}} = it/2, \tau_{\mathcal{M}} = it/2 + 1/2$) that naturally enter the definition of the amplitudes in the direct channel. These rescalings are necessary in order for the amplitudes in the transverse channel to be expressed in terms of the common length of the tube $\ell = -\frac{1}{2\pi} \log q$. The coefficients Γ^i and B_a^i should then be interpreted as the reflection coefficient (one-point function) on a crosscap and on a boundary of type a , respectively. They are related to the integer coefficients in the direct channel by suitable modular transformations:

$$K^i = \sum_j S_j^i \Gamma^j \Gamma^j$$

³We will always omit the modular invariant torus contribution of the parent closed-string theory. Being anomaly free it is irrelevant for our present purposes.

⁴Here and in the following, we will not distinguish between “complex” (unitary) and “real” (orthogonal or symplectic) Chan-Paton multiplicities n^a . The sum over a will include two contributions for the former and only one for the latter, thus producing not only the correct dimensions of the representations but also the correct orientation of the boundary.

$$\begin{aligned}
A_{ab}^i &= \sum_j S_j^i B_a^j B_b^j \\
M_a^i &= \sum_j P_j^i \Gamma^j B_a^j
\end{aligned} \tag{2.3}$$

with S the modular matrix in the character basis and $P \equiv T^{1/2} S T^2 S T^{1/2}$.

In D (even) non-compact dimensions, anomalies are associated to one-loop string amplitudes in the odd-spin structure involving a total number of $D/2 + 1$ graviton and/or gauge field insertions (see [7, 8, 9, 4] for details). One of the vertices has to be taken with longitudinal polarization. The results can be packaged into an anomaly generating polynomial, whose three contributions read [4]

$$\begin{aligned}
\mathcal{K}_{odd} &= -\frac{1}{2} \sum_i \mathcal{I}_i K^i I_A(R) \\
\mathcal{A}_{odd} &= \frac{1}{4} \sum_{i,a,b} \mathcal{I}_i A_{ab}^i ch_{n^a}(F) ch_{n^b}(F) I_{1/2}(R) \\
\mathcal{M}_{odd} &= \frac{1}{4} \sum_{i,a} \mathcal{I}_i M_a^i ch_{n^a}(2F) I_{1/2}(R)
\end{aligned} \tag{2.4}$$

Sum over repeated indices is always understood, unless differently stated. $ch(F)$ is the Chern character of the gauge fields. $I_A(R)$ and $I_{1/2}(R)$ represent the contributions to the gravitational anomaly of a self-dual antisymmetric tensor and a complex spin 1/2 L-fermion, respectively. The important point is that since vertex insertions are aligned along the spacetime directions the internal theory enters only through its partition function in the odd spin structure denoted by \mathcal{I}_i and known as the Witten index. Being topological \mathcal{I}_i is an integer independent of the parameters of the theory and in particular of the worldsheet t -modulus. The additional factor of one-half in the Annulus and Möbius-strip amplitudes reflects the fact that they are counting real fermions. The relation between spin and statistics is responsible for the extra minus sign of the Klein-bottle contribution with respect to the Annulus and Möbius strip. The former can only contribute loops of (anti)self-dual antisymmetric tensors while the latter can only contribute fermionic loops.

The consistency of the string background requires that irreducible terms $\text{tr} R^{\frac{D}{2}+1}$, $\text{tr}_{n^a} F^{\frac{D}{2}+1}$ in the expansion of (2.4) cancel. Cancellations of gauge anomalies imposes a set of conditions on the coefficients A_{ab}^i , M_a^i , while the absence of gravitational anomalies leads to an additional constraint on K^i , A_{ab}^i , M_a^i . Saturating (2.3) with \mathcal{I}_i and using modular invariance of the Witten index, $\mathcal{I}_i S_j^i = \mathcal{I}_j$ and $\mathcal{I}_i P_j^i = \mathcal{I}_j$, one can translate the conditions arising from (2.4) in terms of closed

string variables. After some manipulations, one can see that cancellation of gauge anomalies requires

$$\mathcal{I}_i (2^{D/2} \Gamma^i + B_a^i n^a) = 0 \quad (2.5)$$

where the index “i” is not summed over. This precisely reproduces all tadpole cancellation conditions for massless R-R closed string states belonging to sectors with non-vanishing Witten index $\mathcal{I}_i \neq 0$. It is amusing to observe that once this condition is satisfied irreducible gravitational anomalies are automatically cancelled, i.e. in a consistent open string descendant the absence of irreducible gauge anomalies always implies the absence of irreducible gravitational anomalies. We conclude that: *There is a one-to-one correspondence between the conditions for cancellation of gauge anomalies and RR-tadpoles associated to sectors of the internal SCFT with non-vanishing Witten index.* Tadpoles associated to sectors in the SCFT with vanishing Witten index can be related to “higher dimensional anomalous amplitudes” or to gauge anomalies living on D-brane probes of the vacuum geometry [12]. The early results [10, 11] and the systematic studies [13, 14] represent important intermediate steps to the very general final result.

3 NS-NS tadpoles and RG-flows

The correspondence between anomalies and RR tadpoles apply to any consistent string vacuum configuration containing open and unoriented strings, independently of the presence of any unbroken spacetime supersymmetry. In supersymmetric cases, we can go a step further and consider CP-even couplings that still admits equivalent gauge/supergravity descriptions. Intuitively this should be clear. The sum over even spin structures translates via the “abstruse” ϑ -identity into an odd spin structure contribution to which our previous arguments apply. Let us consider for example $\mathcal{N} = 2$ supersymmetric gauge theories in $D = 4$ dimensions living on N D3-branes in the presence of a background of D7-branes and O7-planes. The running of the four dimensional gauge coupling constant is logarithmic. The coefficient can be extracted from a two-point string amplitude involving two open strings ending on the D3-branes [15, 16]. The result can be written as [5]

$$\beta = \frac{1}{4}(A_{ab}^i n^b + 2M_a^i)h_i = \frac{1}{2}l_2(\mathcal{R}_i^a)h_i \quad (3.1)$$

with $Tr_{\mathcal{R}_i} T^a T^b = l_2(\mathcal{R}_i^a)\delta^{ab}$ and $h_i = -\frac{11}{3}n_V^i + \frac{1}{6}n_O^i + \frac{4}{3}n_F^i$ the standard field theory contribution to the β function of n_O^i scalars, n_V^i vectors and n_F^i Dirac fermions. As usual “i” labels the (massless) characters of the spectrum. In the closed string channel the logarithmic running is associated to massless exchange

in the two overall transverse directions and the coefficient can be matched with (3.1). Indeed noticing that h^i coincides in $\mathcal{N} = 2$ with \mathcal{I}_i one can use (2.3) and $\mathcal{I}_i S^i_j = \mathcal{I}_j$, $\mathcal{I}_i P^i_j = \mathcal{I}_j$ to translate (3.1) into the closed string result

$$\beta = 2(2^{-3}B_b^i n^b + \frac{1}{4}\Gamma^i)B_a^i h_i = 2\hat{T}^i B_a^i h_i \quad (3.2)$$

with $\hat{T}^i = (2^{-3}B_b^i n^b + \frac{1}{4}\Gamma^i)$ the tadpole associated to the character “ i ” and localized at one of the 4 O7-planes. It is worth stressing that although $\mathcal{I}_i = h^i$ the two types of divergences have quite different origins. A careful look at the linear combination of tadpoles appearing in (3.2) reveals that RG-flows is induced by NS-NS tadpoles, and more precisely by the ten-dimensional dilaton tadpole, as expected.

Although the above correspondence between RG-flow and NS-NS tadpoles strongly relies on the presence of $\mathcal{N} = 2$ supersymmetry, some extension is still possible. One can consider D3-branes probing some “F-theory” backgrounds that admit a description in terms of O7-planes and D7-branes at angles. Since an open string with one end on the D3-branes can at most have the other end on one set of D7-branes, states contributing to the β -functions organize themselves into supermultiplets of an $\mathcal{N} = 2$ supersymmetry that is generically broken by the remaining sets of branes. At least to leading order in $1/N$, the correspondence is not expected to be spoiled by higher loop corrections. The potentially dangerous contributions come from worldsheets where all except one of the boundaries lie on the D3-brane. Again an underlying $\mathcal{N} = 2$ supersymmetry prevents their onset. In the following table we list several examples of $\mathcal{N} = 1, 2$ brane configurations where equivalent gauge/supergravity descriptions of the β function coefficients were discussed in [5]

Background	Gauge Group	β_N
D7	$U(N) \times U(n_A) \times U(n_B)$	$n_A + n_B$
D7-O7	$Sp(N) \times SO(n_A)$	$\frac{1}{2}(n_A - 8)$
C^4/Z_2	$U(N) \times U(n_A)$	$\frac{1}{2}(n_A + \bar{n}_A - 8)$
$C^6/Z_2 \times Z_2$	$Sp(N) \times Sp(n_A) \times Sp(n_B) \times Sp(n_C)$	$\frac{1}{2}(n_A + n_B + n_C - 12)$

From the table one can easily identify the values of n ’s for which the background is conformal, *i.e.* $\beta_N = 0$.

Finally we would like to briefly put the above results in the perspective of the AdS/CFT correspondence. The presence of a dilaton tadpole can be seen as a perturbation of the supergravity field equations from the AdS configuration (conformal point). Fortunately the perturbed equation for the dilaton can be explicitly solved [5] in the large N limit and one is left with a logarithmically running (in the radial coordinate). The coefficient is precisely given by the dilaton

tadpole in agreement with our previous analysis. Similar results for D3-branes probing a type 0 background were originally found in [17].

Acknowledgements

We would like to thank the participants to the Meeting for fruitful and stimulating discussions. This work is partly supported by the PPARC grant, by the EEC contract HPRN-CT-2000-00122, and by the INTAS project 991590.

References

- [1] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. **D55** (1997) 5112, hep-th/9610043.
- [2] J.M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231; Int. J. Theor. Phys. **38** (1999) 1113, hep-th/9711200.
- [3] J. F. Morales, C. A. Scrucca and M. Serone, Phys. Lett **B417** (1998)233, hep-th/9709063 Nucl. Phys. **B534** (1998) 223 [hep-th/9801183];
- [4] M. Bianchi and J.F. Morales, **JHEP** **0003** (2000) 030, [hep-th/0002149]
- [5] M. Bianchi and J.F. Morales, **JHEP** **0008** (2000) 035, [hep-th/0006176]
- [6] A. Sagnotti, in *Non Perturbative Quantum Field Theory*, ed. G. Mack et al. (Pergamon, New York, 1988) 521; M. Bianchi, G. Pradisi and A. Sagnotti, Phys. Lett. **B273** (1991) 389; D. Fioravanti, G. Pradisi and A. Sagnotti, Phys. Lett. **B321** (1994) 349; G. Pradisi, A. Sagnotti, Ya.S. Stanev, Phys. Lett. **B354** (1995) 279; **B356** (1995) 230; **B381** (1996) 97, [hep-th/9603097]. M. Bianchi and A. Sagnotti, Phys. Lett. **B247** (1990) 517; Nucl. Phys. **B361** (1991) 519.
- [7] T. Inami, H. Kanno and T. Kubota, Phys. Lett. **B199** (1987) 389; Nucl. Phys. **B308** (1988) 203.
- [8] C.A. Scrucca and M. Serone, Nucl.Phys. **B564** (2000) 555, hep-th/9907112.
- [9] J.F. Morales, C.A. Scrucca and M. Serone, Nucl. Phys. **B552** (1999) 291 [hep-th/9812071];
- [10] Y. Cai and J. Polchinski, Nucl. Phys. **B296** (1988) 91.

- [11] N. Marcus and A. Sagnotti, Phys. Lett. **B188** (1987) 298; M. Bianchi and A. Sagnotti, Phys. Lett. **B211** (1988) 298, Phys. Lett. **B231** (1990) 389; G. Pradisi and A. Sagnotti, Phys. Lett. **B216** (1989) 59.
- [12] A.M. Uranga, *D-brane probes, RR tadpole cancellation and Ktheory charge*, hep-th/0011048.
- [13] J.D. Blum and K. Intriligator, Nucl. Phys. **B506** (1997) 223, hep-th/9705030; R.G. Leigh and M. Rozali, Phys. Rev. **D59** (1999) 026004, hep-th/9807082.
- [14] G. Aldazabal, D. Badagnani, L.E. Ibanez and A.M. Uranga, JHEP **06** (1999) 031, hep-th/9904071.
- [15] C. Bachas and C. Fabre, Nucl. Phys. **B476** (1996)418, [hep-th/9605028].
- [16] I. Antoniadis, C. Bachas, E. Dudas Nucl. Phys. **B560** (1999)93, [hep-th/9906039].
- [17] C. Angelantonj and A. Armoni, “*Non-Tachyonic Type 0B Orientifolds, Non-Supersymmetric Gauge Theories and Cosmological RG Flow*”, hep-th/9912257; “*RG Flow, Wilson Loops and the Dilaton Tadpole*”, hep-th/0003050.